HW 1-4 (1-3 p 72) Page 1

1.3 (p 72) 63, 65*, 67, 74

Read 1.4 (p 81-82) ✓

1.4 (p 83) 5, 7, 9, 13, 18, 20, 21, 23, 25

Read 1.5 (p 87-91)

Th 1.9 p 89

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$

(63) $\lim_{x\to 0} \frac{\sin x}{5x} = \left(\lim_{x\to 0} \frac{\sin x}{x}\right)\left(\frac{1}{5}\right) = [1]\frac{1}{5} = \frac{1}{5}$

(65) $\lim_{x\to 0} \frac{\sin x\,(1-\cos x)}{x^2}$

$\lim_{x\to 0} \left(\frac{\sin x}{x}\right)\left(\frac{1-\cos x}{x}\right) = (1)(0) = 0$

(67) $\lim_{x\to 0} \frac{\sin^2 x}{x} = \lim_{x\to 0} (\sin x)\left(\frac{\sin x}{x}\right) = (0)(1) = 0$

(74) $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x}$

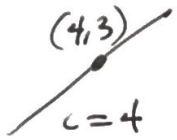
$\lim_{x\to 0} \left(\frac{\sin 2x}{1}\right)\left(\frac{1}{\sin 3x}\right)$

$\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)\left(\frac{x}{\sin 3x}\right)$

$\lim_{x\to 0} \left(\frac{2\sin 2x}{2x}\right)\left(\frac{3x}{3\sin 3x}\right)$

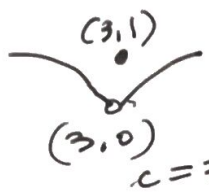
$(2(1))\left(\frac{1}{\frac{3\sin 3x}{3x}}\right)$

$(2)\left(\frac{1}{3(1)}\right) = \frac{2}{3}$

⑤ 

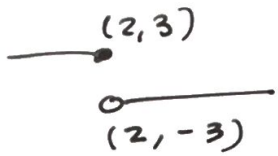
(a) $\lim_{x\to c^+} f(x) = 3$ (b) $\lim_{x\to c^-} f(x) = 3$ (c) $\lim_{x\to c} f(x) = 3$

∴ continuous at $x=4$ and cont. on $(-\infty,\infty)$

⑦ 

(a) $\lim_{x\to c^+} f(x) = 0$ (b) $\lim_{x\to c^-} f(x) = 0$ (c) $\lim_{x\to c} f(x) = 0$

but NOT cont. since $f(3) = 1, \neq 0$

⑨ 

(a) $\lim_{x\to c^+} f(x) = -3$ (b) $\lim_{x\to c^-} f(x) = 3$

(c) $\lim_{x\to c} f(x)$ DNE, f is NOT cont.

⑬ $$\lim_{x\to 5^+} \frac{x-5}{x^2-25} = \lim_{x\to 5^+} \frac{(x-5)}{(x+5)(x-5)} = \frac{1}{5+5} = \frac{1}{10}$$

⑱ $$\lim_{x\to 10^+} \frac{|x-10|}{x-10} = \lim_{x\to 10^+} \frac{x-10}{x-10} = 1$$

⑳ $$\lim_{\Delta x\to 0^+} \frac{(x+\Delta x)^2 + x + \Delta x - (x^2+x)}{\Delta x}$$

$$= \lim_{\Delta x\to 0^+} \frac{(x^2 + 2x\Delta x + (\Delta x)^2 + x + \Delta x - (x^2 - x))}{\Delta x}$$

$$= \lim_{\Delta x\to 0^+} \frac{2x\Delta x}{\Delta x} + \frac{(\Delta x)^2}{\Delta x} + \frac{\Delta x}{\Delta x} = 2x + 0 + 1 = 2x+1$$

㉑ $\lim_{x\to 3^-} f(x)$ where $f(x) = \begin{cases} \frac{x+2}{2}, & x < 3 \\ 12 - 2x, & x > 3 \end{cases}$ ✓

$$= \lim_{x\to 3^-} \frac{x+2}{2} = \frac{3+2}{2} = \frac{5}{2}$$

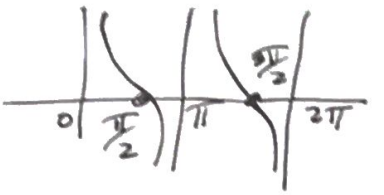
㉓ $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} x^3+1, & x<1 \\ x+1, & x \geq 1 \end{cases}$

$\lim_{x\to 1^-} f(x) = (1)^3 + 1 = 2$

$\lim_{x\to 1^+} f(x) = 1 + 1 = 2$

∴ $\lim_{x\to 1} f(x) = 2$

(25) $\lim_{x \to \pi} \cot x = \lim_{x \to \pi} \frac{\cos x}{\sin x}$



$\lim_{x \to \pi^-} \frac{\cos x}{\sin x} = \frac{-1}{0}$ DNE

$\lim_{x \to \pi^+} \frac{\cos x}{\sin x} = \frac{-1}{0}$ DNE

$\therefore \lim_{x \to \pi} \cot x$ DNE